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Author(s): Heatwole, Eric Mann
Parker, Gary R. Jr.
Dickson, Peter
Holmes, Matthew D.
Gunderson, Jake A.

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Probability Calculation of Grit-Grit Interaction on a Plastic Bonded Explosive During Glancing Impacts

Eric Heatwole, Gary Parker, Peter Dickson, Matt Holmes and Jake Gunderson
WX-6, LANL

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1 Background

Recent experiments indicate that frictionally heated grit can act as effective hot spots for the ignition of PBX 9501 (Parker *et al.* (2013)). The most common mode for hot spot formation is when a piece of grit is dragged by the explosive across a high melting point surface, such as steel. This leads to a hot spot with a temperature of the melting point of the surface. For high melting point surfaces this can be hot enough to lead to ignition. It is believed that an easy way to mitigate this is to use surfaces with low melting points, which means the hot spots formed cannot become hot enough to ignite the explosive during the duration of the impact (Bowden and Gurton (1948)). However, when surfaces have a similar hardness to the PBX (such as many plastics), hot spots of sufficient temperature for ignition can still be formed by the collision of two grit particles, one being dragged by the PBX with the other being stationary on the surface.

In order to mitigate the probability of ignition it is desirable to know how clean the work surfaces need to be in order to minimize the acceptable risk of an event. Two cases need to be considered: first, what is probability that a dropped piece of PBX will contact one or more grit particles? This will lead to an estimate of how "clean" a surface needs to be when working with unmitigated, high-melting point surfaces. Second, what is the probability that a dropped piece of PBX will lead to a collision between two or more grit particles? This will give an estimate how clean a surface needs to be when working with low melting point surfaces.

2 Initial Assumptions

In order to obtain a workable form for the probabilities of interest several assumptions must be made. The first of the assumptions is that the probability of the location of the grit particles is statistically independent from each other. In other words, that the density of the grit is low enough that there will be no overlap of the particles. This assumption allows us to approximate the distribution of grit particles as a Poisson distribution (Yates and Goodman (2005)),

$$f(k, \lambda) = \frac{\lambda^k}{k!} \exp(-\lambda), \tag{1}$$

where k would be the number of particles in the area of interest and $\lambda = \rho A$, where ρ is the average particle density of the surface and A is the area of interest. For example, this states that the probability that there are two particles in some area A with an average particle density, ρ, would be $\frac{\lambda^2}{2} \exp(-\lambda)$. For this case, the area, A, will be the total area swept out by the contacting explosive moving over the impacted surface, Fig.(1) and Fig.(3). This area will depend on several variables, including drop height and the angle of impact. This gives the following expression for the probability that a dropped piece of PBX will impact one or more grit particles:

$$P_a(A, \rho) = \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda}. \tag{2}$$

Since the sum of P_a over all k must be unity, P_a can be written in the more compact form:

$$P_a(A, \rho) = 1 - f(0, \lambda) = 1 - e^{-\lambda}. \tag{3}$$

Where, as before, $\lambda = \rho A$. Figure (2) shows the a plot of P_a versus grit density for a range of contact areas.

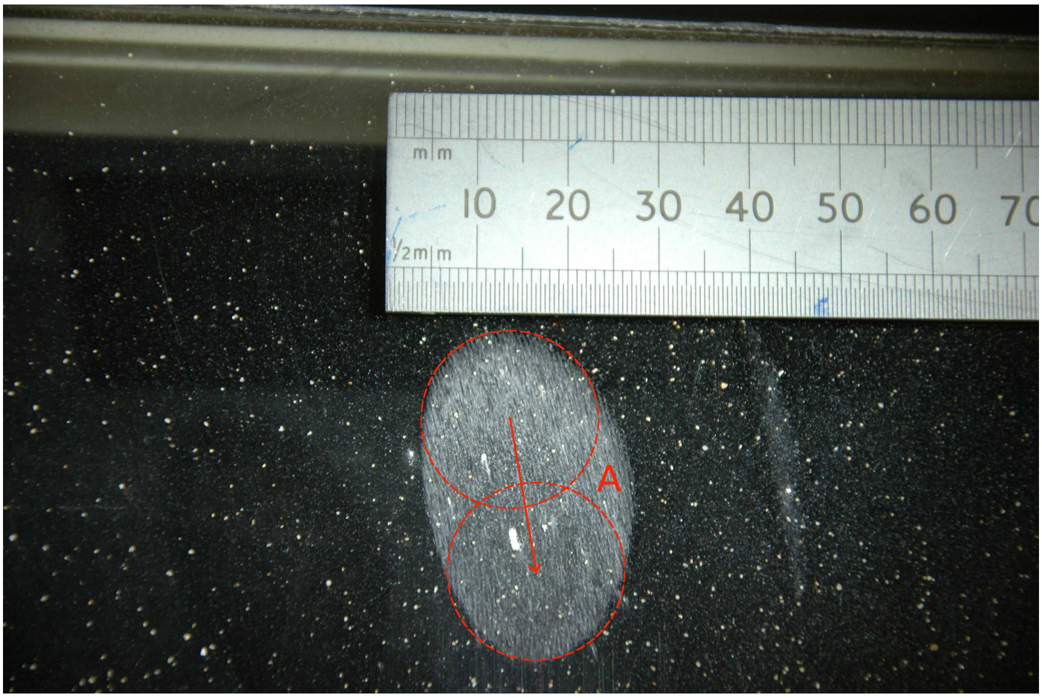

Figure 1: Showing the total area, A, of the swept out by the contacting explosive

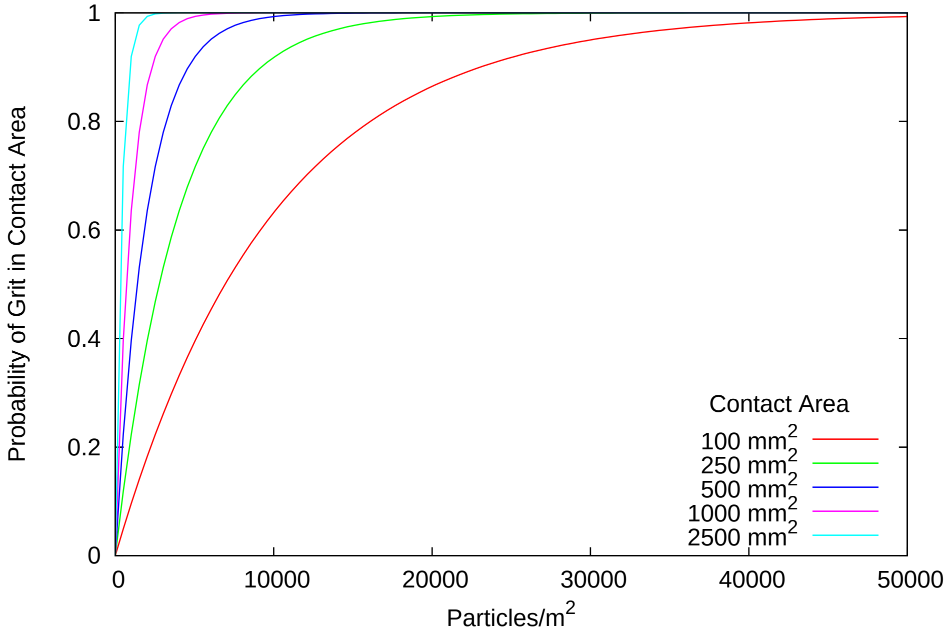

Figure 2: Probability of one more particles within contact area versus grit density

3 Grit-on-Grit

The second calculation determines the probability that a grit-on-grit collision occurs, P_c, and builds on the first calculation. Starting with the assumptions from before, the probability of having two or more particles in the impact area (less than two particles and no collision can occur) can be written as:

$$P_{2a}(A, \rho) = \sum_{k=2}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda}. \tag{4}$$

If there are k particles in the impact area, then, assuming each particles in the area has an equal probability of moving, p, the probability of having j out of k particles moving is:

$$\left(\begin{array}{c} k \\ j \end{array} \right) p^j \, (1-p)^{k-j} \, . \tag{5}$$

The probability, p, depends on the material characteristics and other impact parameters and it's exact determination is outside of the scope of this conversation. However, it is possible to estimate the probability of moving by considering the material relative hardness, if the substrate is much harder than the PBX then all of the particles will move and if the the substrate has a similar hardness to the PBX then the probability of moving will be ≈ 0.5. Also, as will be shown later, for $0.1 < p < 0.9$, the probability of having a grit-grit collision is not strongly influenced by p. So the probability of one or more grit particles moving can be written as:

$$P_M = \sum_{k=2}^{\infty} \sum_{j=1}^{k-1} \frac{\lambda^k}{k!} e^{-\lambda} \left(\begin{array}{c} k \\ j \end{array} \right) p^j (1-p)^{k-j}. \tag{6}$$

The bounds in the second sum arise from the fact that in order for a collision to occur there needs to be at least one moving particle or one stationary particle. In the bounding cases, that all the particles are stationary or all the particles are in motion are included then the bounds would be $j = 0, k$ respectively.

Now to finish the derivation, the probability that a collision occurs, if there are k particles in an area A, with j particles moving and $k-j$ stationary particles, needs to be calculated. To do so several approximations need to be made. First, it is assumed that all particles have the same size, i.e. all of the particles have the same radius. This allows the moving particles to be treated as having a radius of $2r$ while the stationary particles are treated as point particles. This means if a particle moves some distance x, then it sweeps out the area $4rx$, where the hemi-spherical ends can be neglected. Therefore, the probability that any one stationary particle in the contact area, A, is also in the area swept out by the moving particle is $\frac{4rx}{A}$, since the stationary particles are equally likely to be anywhere in the area A. The probability that i particles out of $k-j$ are in the area swept out by the moving particle is given as,

$$\begin{pmatrix} k-j \\ i \end{pmatrix} \left(\frac{4rx}{A}\right)^{i} \left(1-\frac{4rx}{A}\right)^{k-j-i}. \tag{7}$$

So the probability that one or more particles are in the area swept out is,

$$1-\left(1-\frac{4rx}{A}\right)^{k-j}. \tag{8}$$

The next approximation has to do with the distance the particles can move. As the explosive impacts the surface, the initial area contacting the surface can be approximated as a circle of radius R, and it is assumed that this contact area does not change as the explosive slides over surface; in other words the radius of the circle does not change. The center of the circle moves some distance L as it slides over the surface, which gives the total area of contact,

$$A_{tot} = 2RL + \pi R^2. \tag{9}$$

This means that any moving particle trapped in the sliding explosive can move a maximum distance of L. Therefore, any moving grit particle can be displaced any distance between zero and L, and it is approximated that it is equally likely it will start moving anywhere within this constraint. Now since the particles are assumed to be statistically independent, it can be approximated that each moving grit particle starts moving in the rectangle of area $\acute{A} = LW$ (see Fig.(3)) where $W = 2R$ and the particle is equally likely to start moving anywhere in this rectangle. This means that the probability that a moving grit particle starts moving at x can be written as $\frac{dx}{L}$. Combining this with Eq. (8) and integrating over x leads to the probability, P_c, that a

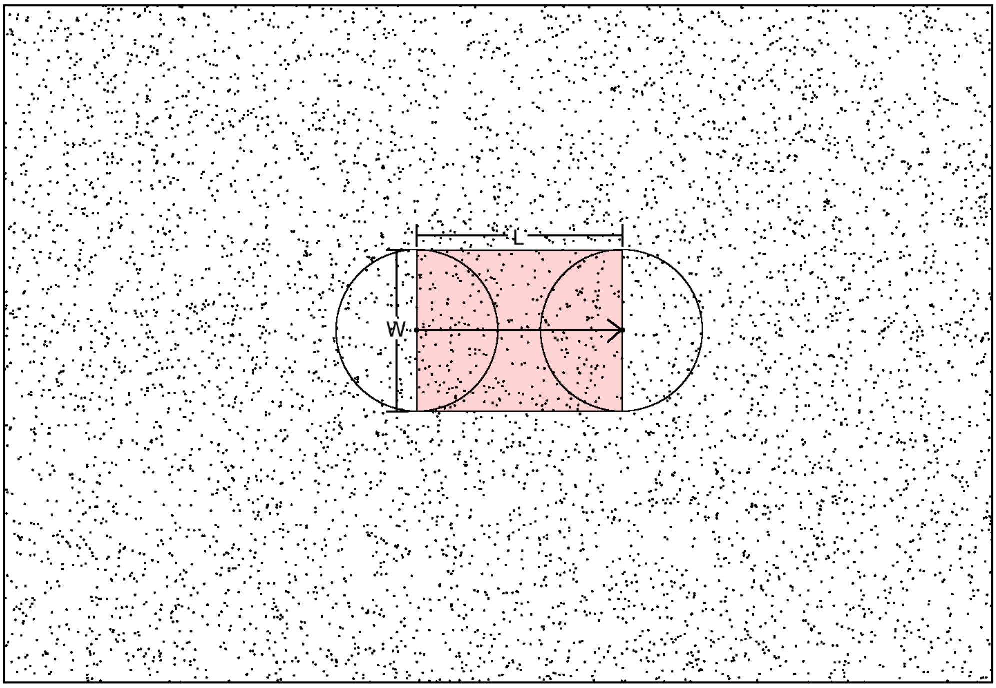

Figure 3: Illustrating the area swept out by an impacting circle. The particles are shown in this diagram with a Poisson distribution.

single moving particle collides with one or more stationary particles:

$$P_c = \int_0^L \left(1 - \left(1 - \frac{4r}{A}x\right)^{k-j}\right) \frac{dx}{L}. \tag{10}$$

The integral in Eq.(10) can be solved via substitution to give the following result for P_c,

$$P_c = 1 + \frac{A}{4rL(k-j+1)} \left\{ \left(1 - \frac{4rL}{A}\right)^{k-j+1} - 1 \right\}. \tag{11}$$

Since each moving particle is assumed to be independent the probability of each of the i moving particles out of j having at least one collision is,

$$\left(\begin{array}{c} j \\ i \end{array} \right) P_c^i \left(1 - P_c\right)^{j-i}, \tag{12}$$

and the probability probability of j moving particles having one or more collisions is,

$$1 - \left(1 - P_c\right)^j. \tag{13}$$

Now combining Eq.(6) and Eq.(13) such that each term in Eq.(6) is weighted by Eq.(13) gives the total probability, P_{tot} that a grit-grit collision occurs,

$$P_{tot}(A, L, \rho, p, r) = \sum_{k=2}^{\infty} \sum_{j=1}^{k-1} \frac{\lambda^k}{k!} e^{-\lambda} \left(\begin{array}{c} k \\ j \end{array} \right) p^j \left(1 - p\right)^{k-j} \left(1 - \left(1 - P_c\right)^j\right), \tag{14}$$

or, in its full form:

$$\begin{aligned} &P_{tot}(A, L, \rho, p, r) = \\ &\quad \sum_{k=2}^{\infty} \sum_{j=1}^{k-1} \frac{\lambda^k}{k!} e^{-\lambda} \left(\begin{array}{c} k \\ j \end{array} \right) p^j \left(1 - p\right)^{k-j} \\ &\quad \left(1 - \left\{ \frac{A}{4rL\left(k-j+1\right)} \left(1 - \left(1 - \frac{4rL}{A}\right)^{k-j+1}\right) \right\}^j \right). \end{aligned} \tag{15}$$

4 Estimation and Calibration of the Impact Area

In this section we derive an approximation for the area and skid length for an impacting hemisphere of 9501. Both the area and skid length are dependent on the drop height of the hemisphere and will be calibrated from experiment.

The expression for the contact area is determined using a combination of theory and experiment. The skid experiment used a rigid-arm pendulum (See Fig.(4)) to deliver HE charges onto an impact surface with good accuracy and reproducibility. The HE charges used were live-pole hemispheres, which are a two part system consisting of a hemisphere of Delrin with an insert of PBX 9501 which impacts with the target. See (Fig. (5)).

This hemi is delivered onto the impact surface mounted on a rigid target frame by the pendulum. For the initial set of experiments the angle between the HE charge and the target was set at 45^o. The orientation of the impact surface and target frame allows for optical access on the normal axis, allowing the contact surface of transparent targets to be observed. This allowed the contact area to be measured as the impact occurred by the use of high-speed video cameras (see Fig.(6)). For a more detailed explanation of the experimental setup please see (Parker *et al.* (2013)).

In order to apply the proceeding derivation to the case of grit mediated ignition in an explosive, we must obtain an expression for the area, A and slide length, L, using reasonable approximations. While the final form for the time dependent area of the impacting PBX is flawed, it is shown that the maximum impact area fits the experimental data (see Fig. (8)) in a reasonable fashion. This allows us to find a usable form for A and L for use in Eq. (15) which depends on both velocity (or equivalently drop height) and impact angle which allows us to explore the effect of both these parameters. For the derivation several approximations are made, including the first and most problematic, that the forces on the hemisphere are high enough that the material response is in the plastic regime. As such, that the stress can be approximated as a constant. A discussion on why this approximation fails but leads to valuable physical insights will occur after the outline of the

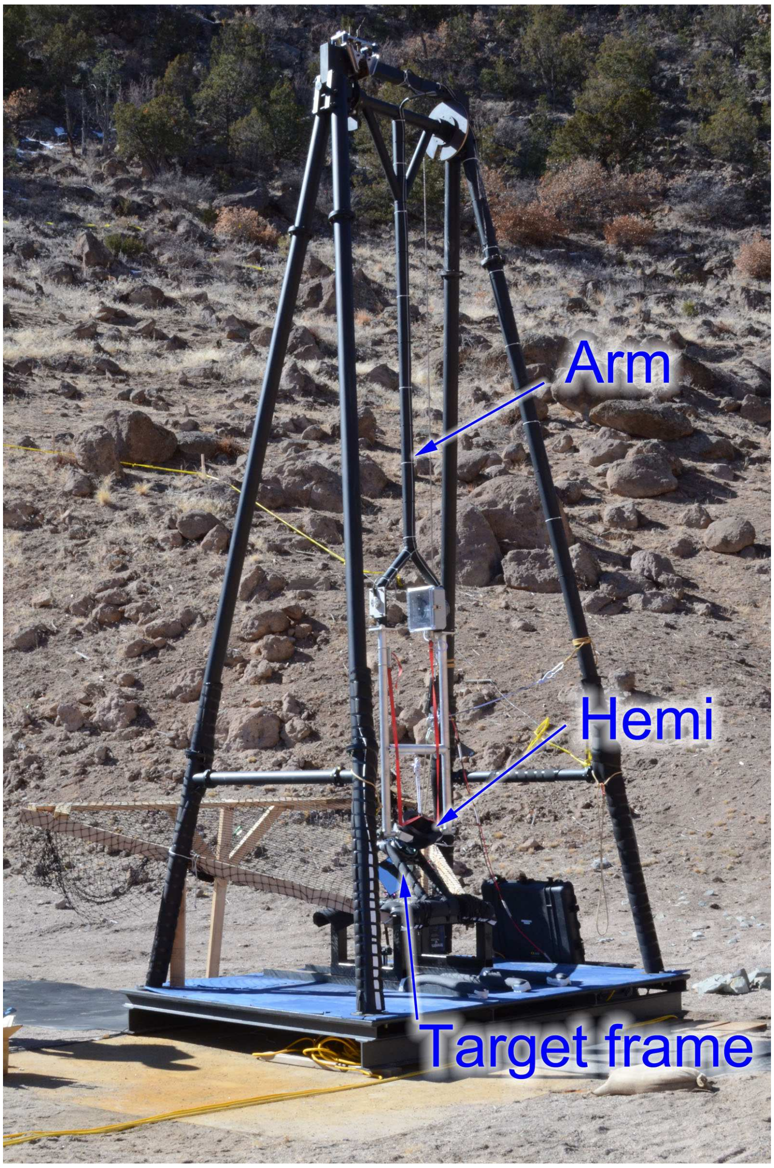

Figure 4: Overview of the pendulum used for skid testing

derivation. This leads to the expression,

$$\frac{F}{A} = -\alpha, \tag{16}$$

which states that the total force of the impacting hemisphere is proportional

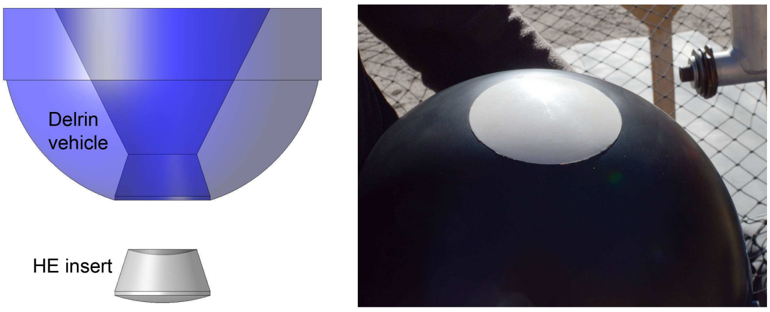

Figure 5: Showing delrin holder with PBX 9501 insert.

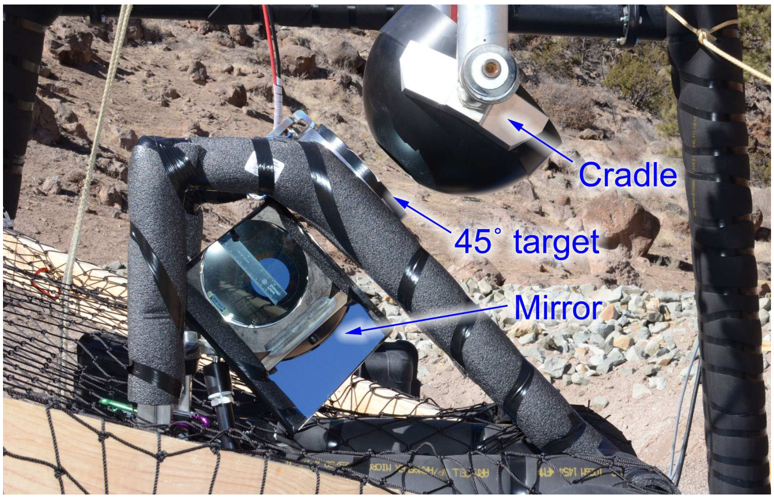

Figure 6: Showing setup of target with optical path

to the area in contact with the substrate. When the hemisphere impacts the target, the impacting surface of the hemisphere starts to compress, we define the amount of compression from the hemisphere's initial equilibrium point to be the coordinate x. As the hemisphere compresses, the circular area in contact with the substrate is dependent on how far the hemisphere is compressed, or equivalently, x. It is simple to derive an expression for the area of contact which depends on x and the radius of curvature, R_0, of the hemisphere from a geometric argument (See Fig. (7)). This leads to an expression for $A(x)$ as,

$$A(x) = 2\pi R_0 x - \pi x^2. \tag{17}$$

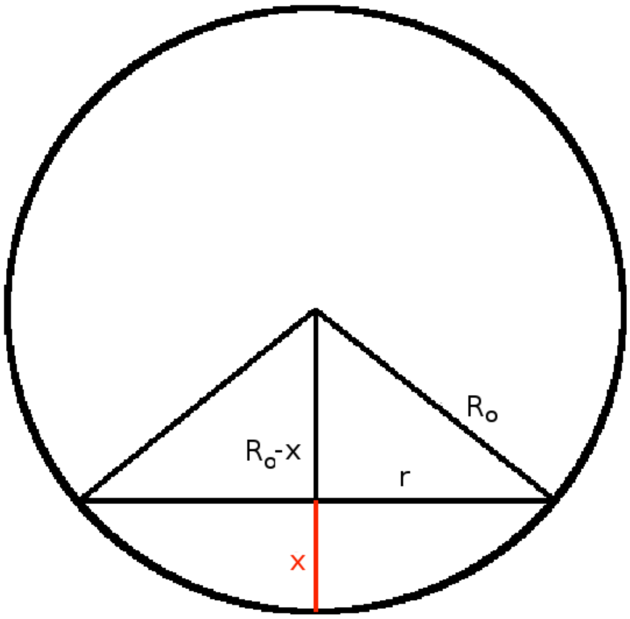

Figure 7: Two dimensional representation showing the the crucial geometry for determining the contact area with respect to compression of the hemisphere. The radius of the contact area, r, is determined using the Pythagorean theorem, while x is the compression coordinate used in the resulting differential equation.

We can then use the equation for the area from Eq. (16), which leads to,

$$F = -\alpha A(x) = -\alpha\pi\left(2R_0x - x^2\right), \tag{18}$$

which gives the differential equation with respect to our coordinate, x, as,

$$\frac{d^2x}{dt^2} = -\frac{\alpha\pi}{M}\left(2R_0x - x^2\right), \tag{19}$$

where M is the mass of the hemi. Since x is very small compared to the radius of curvature, R_0, the x^2 term in the proceeding differential can be dropped, which leads to following approximation,

$$\frac{d^2x}{dt^2} \approx -\frac{\alpha 2\pi R_0}{M}x = -\omega^2 x. \tag{20}$$

Readers will recognize this as the equation of motion for a harmonic oscillator, where $\omega = \sqrt{\frac{\alpha 2\pi R_0}{M}}$. The initial conditions for the system are then $x_0 = 0$ and $p_0 = Mv_0$, the initial displacement and momentum respectively, which leads to the time dependent equation for A as,

$$A(t) = 2\pi R_0x(t) - \pi x(t)^2 \approx 2\pi R_0x(t) = \frac{2\pi R_0v_0}{\omega}sin(\omega t). \tag{21}$$

Due to the complex physics occurring in an actual drop and the rather gross level of approximations made, including the aforementioned plastic deformation, as well as the assumption that the pressure is constant throughout the entire impact area, it is not possible to fit Eq.(21) to the experimental time dependent area. While Eq.(21) fails to adequately describe the time evolution of the contact area, it does capture one useful characteristic, that the maximum area, A_{max}, scales linearly with the impact velocity. It is straightforward to obtain the equation of motion for the elastic limit rather than plastic limit, in a similar manner in which Eq.(19) was obtained. The derivation of this equation of motion is left as an exercise for the reader. A numerical exploration showed that the maximum area scales as $\approx v_0^{0.666}$ rather than scaling linearly as in the plastic case, suggesting that while the simplified model does not capture the correct time evolution but the plastic deformation is nevertheless the dominant process.

$$A_{max} = \frac{2\pi R_0 v_0}{\omega} \tag{22}$$

Eq.(22) can then be fit to the empirically determined maximum area data versus impact velocity using ω as the fitting parameter (which is equivalent to fitting α). This fit was done using the measured radius of curvature of the PBX 9501 as $R_0 = 0.127$ m and a total mass, $M = 6.8$ kg. The fit gives a value of $\omega = 7860$ s^{-1} which is equivalent to $\alpha = 0.53$ MPa. This value for α is approximately an order of magnitude lower than the yield strength of PBX 9501 (Dobratz (1985)). This seems to imply that the yield strength of the PBX is never reached, therefore the deformation should be elastic not plastic but the measured ω is the averaged value for the entire contact area. This fit gives an expression for the maximum area (See Fig.(8)) which can be used in determining the total area for use in Eq.(15). This area is slightly underestimated due to the frame rate of the camera used to collect the area data, but with the appropriate time dependent expression for the area, the maximum area could be better fit. However, that is currently beyond the scope of this report.

Next, the length of the skid needs to calculated. Since the experimental evidence indicates that the time of the bounce ($t_b \sim 1ms$) is largely independent of the drop height, the length of the skid can be well approximated as the bounce time multiplied by the impact velocity in the direction of the skid. This allows us to write an expression for the total area, A_{tot} and length,

$$\begin{aligned} A_{tot} &= sin\theta\frac{2\pi R_0 v_0}{\omega} + 2t_b cos(\theta) v_0^{\frac{3}{2}}\sqrt{\frac{2R_0 sin(\theta)}{\omega}} \\ L &= t_b v_0 cos(\theta) \end{aligned} \tag{23}$$

Where θ is the angle of the incident impact.

5 Results

The final form for the total probability of a grit-on-grit collision, Eq.(15), depends on five variables; A (the area), L (the distance the explosives slides over the surface), ρ (the average density of grit particles on the surface), p (the probability the grit particle moves) and r (the radius of the grit particles, assumed to be equal for all particles). Of these, the area, A and the length L are dependent on the drop height and angle of impact and Eq.(23) will be used

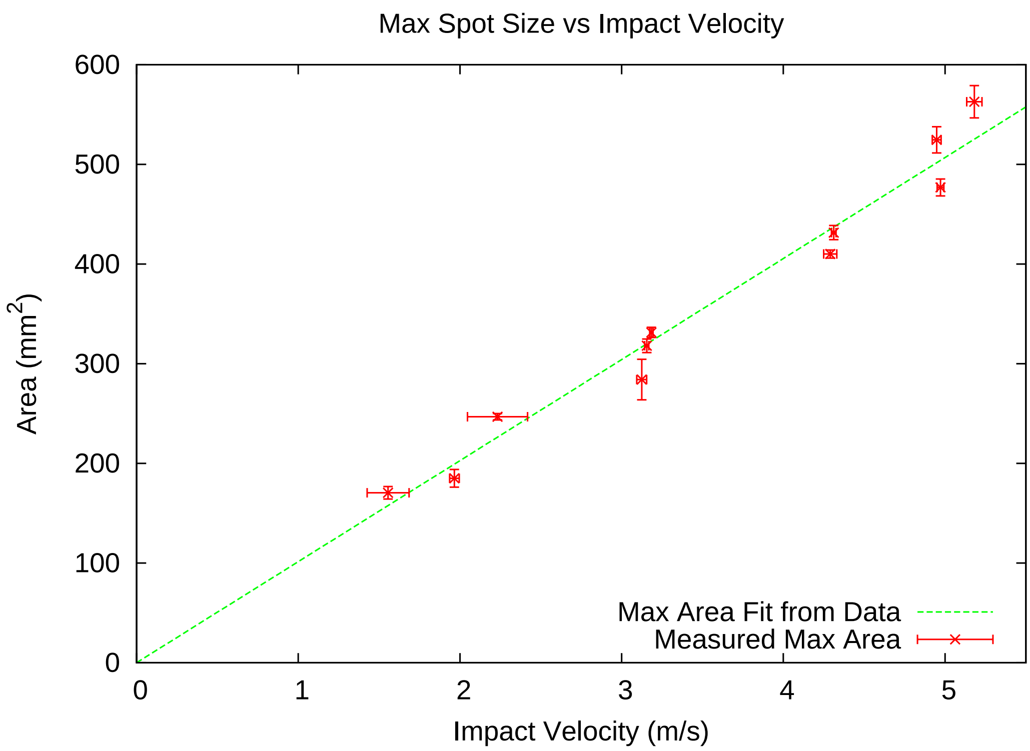

Figure 8: Plot showing measured maximum area versus impact velocity and the fit.

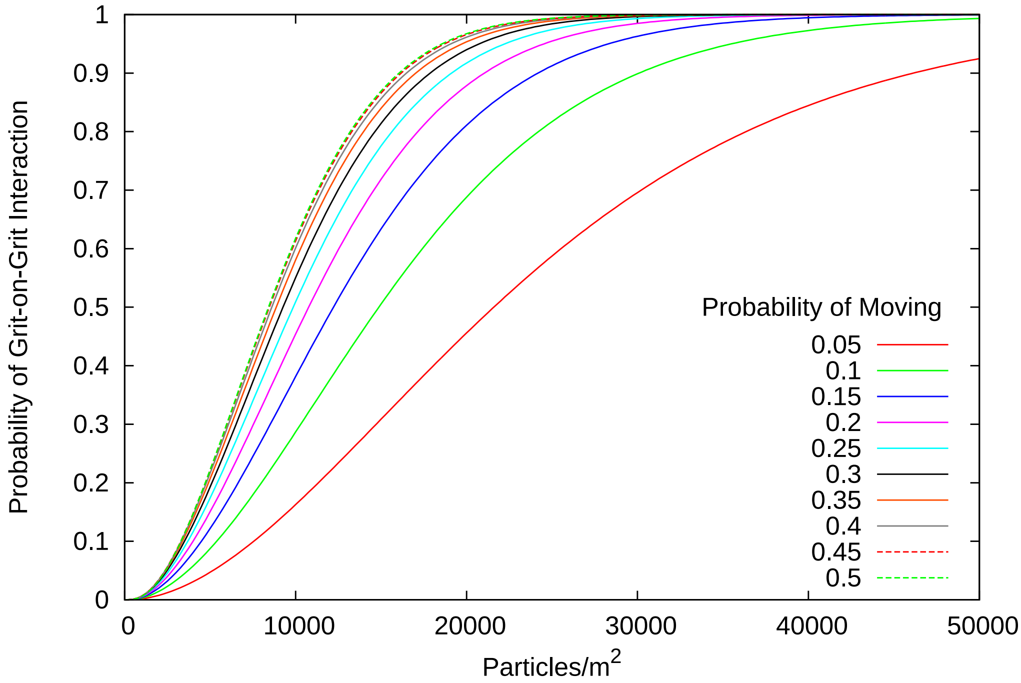

Figure 9: A plot showing the dependence of P_{tot} on p for a range of values for p

as a functional form for these quantities. The coverage density and radius of grit particles, ρ and r, respectively, are taken to be the independent variables for this analysis, since these give a measure of the "cleanliness" needed to minimize the event of a reaction. The other variable, p, the probability of a grit particle moving does not strongly affect the overall probability of a collision except in cases of low (<0.1) and high (>0.9) probabilities of the grit moving. This is illustrated in Fig.(9) and Fig.(10).

Considering the experimental setup described above, with $\theta = 45^o$, we can rewrite Eq.(23) with respect to drop height, since $v_0 = \sqrt{2gh}$, as

$$A_{tot} = \frac{2\pi R_0\sqrt{gh}}{\omega} + 2t_b\left(gh\right)^{\frac{3}{4}}\sqrt{\frac{2R_0}{\omega}} \qquad (24)$$
$$L = t_b\sqrt{gh}.$$

Where h is the drop height and g is the acceleration due to gravity. Using

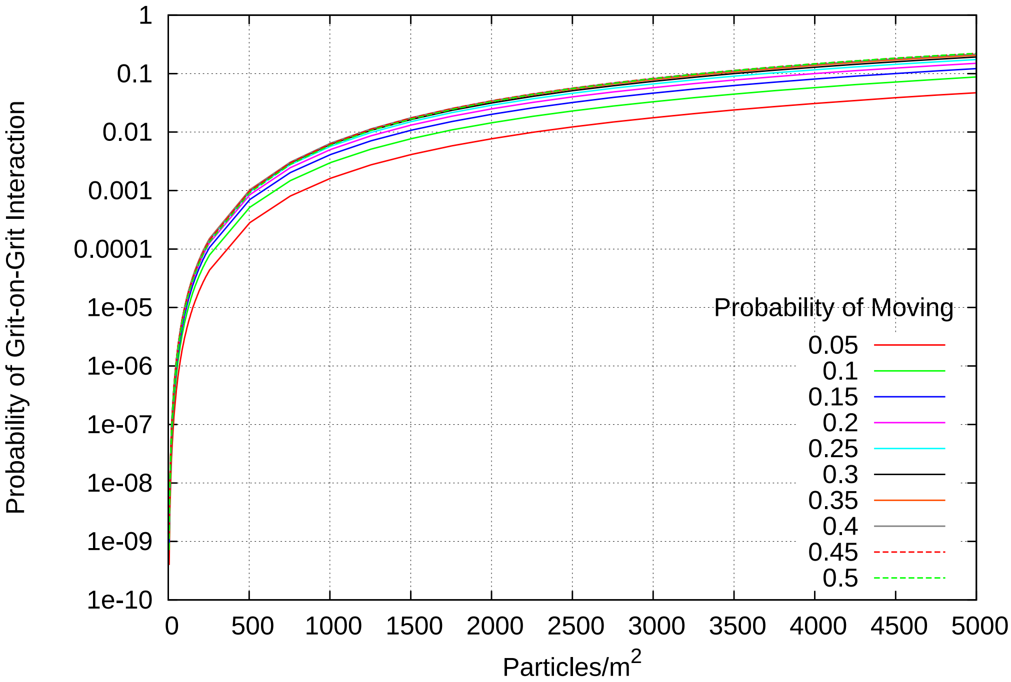

Figure 10: A log plot showing the dependence of P_{tot} on p for a range of values for p

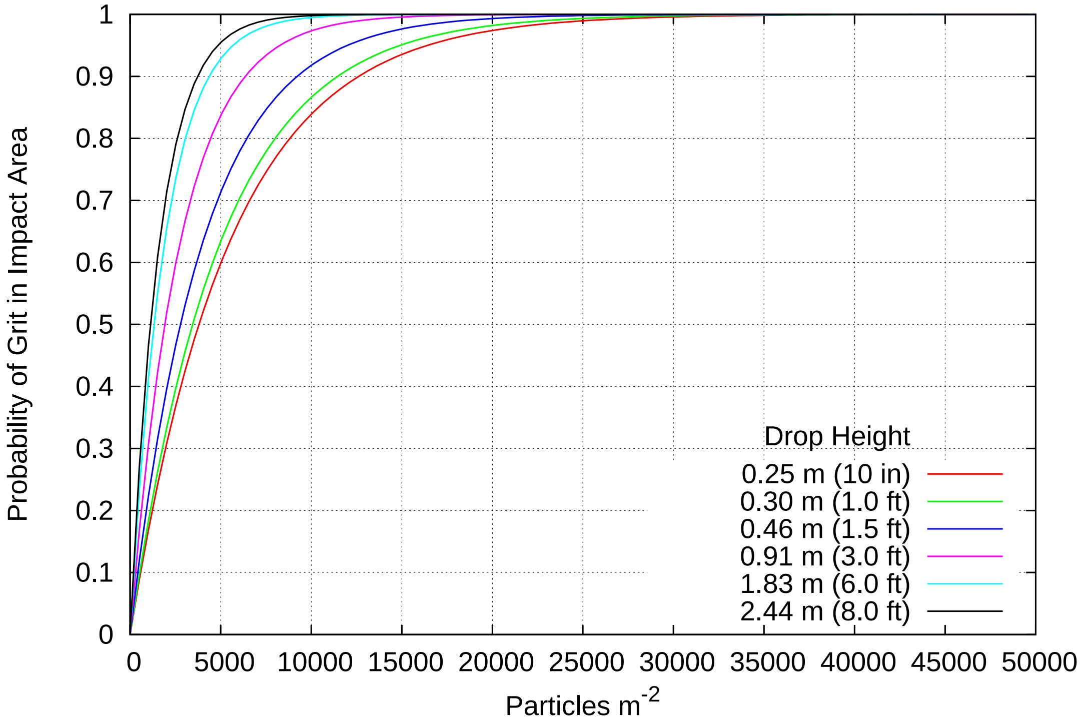

Figure 11: A plot showing the dependence of the probability of impacting one or particles, P_{tot} on drop height and coverage density, ρ. The fixed parameters were particle size, $r = 250\ \mu m$ and impact angle, $\theta = 45^o$
.

the measured parameters $R_0 = 0.127$ m and $t_b = 1\ ms$ with the fit parameter $\omega = 7860 sec^{-1}$, this allows us to calculate the probability that one or more particles of grit will be within the impact area, depending on drop height and coverage density (See Fig.(11)). We can also calculate the probability of grit-grit collision with the preceding parameters. For this case we will pick the probability of moving to be $p = 0.50$, which is representative of the target surface having a similar hardness to PBX 9501. See Fig.(12)).

We can explore the effect of changing the impact angle on both the probability that one or more particles are in the impact area (Fig.(13)) and the probability of a grit-grit interaction (Fig.(14)). For both cases a 1.83 m (6 ft.) drop was used, with a particle radius of 250 μm. The probability of mov-

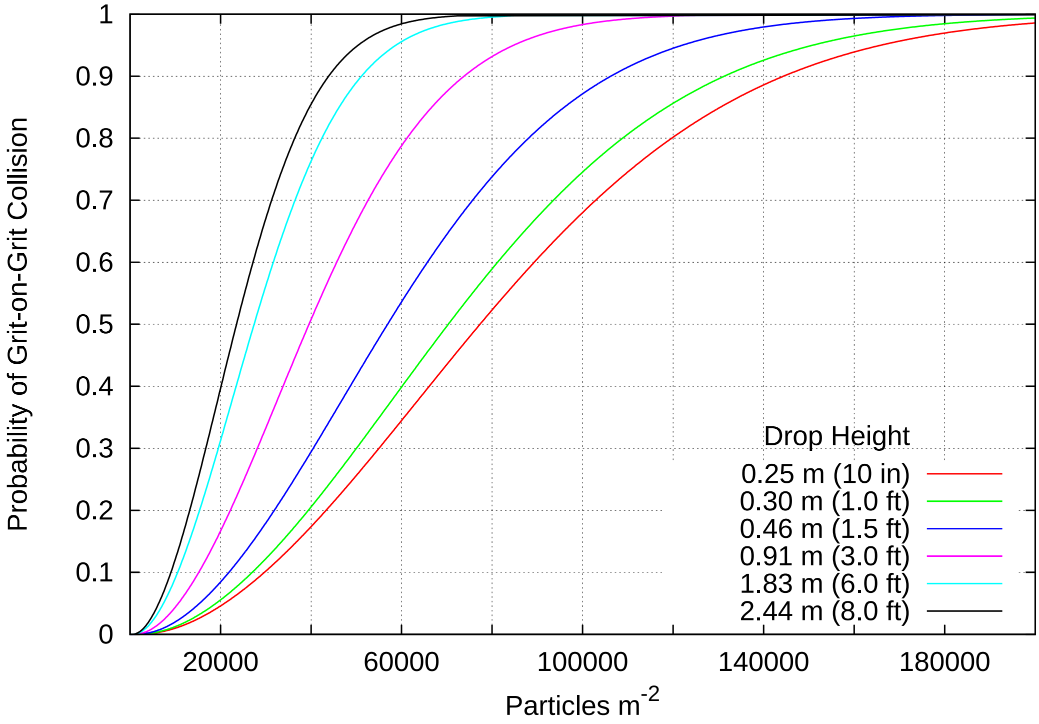

Figure 12: A plot showing the probability of grit-grit interaction depending on drop height and coverage density, ρ. The fixed parameters were particle size, $r = 250\mu m$, probability of moving, $p = 0.50$, and impact angle, $\theta = 45^o$

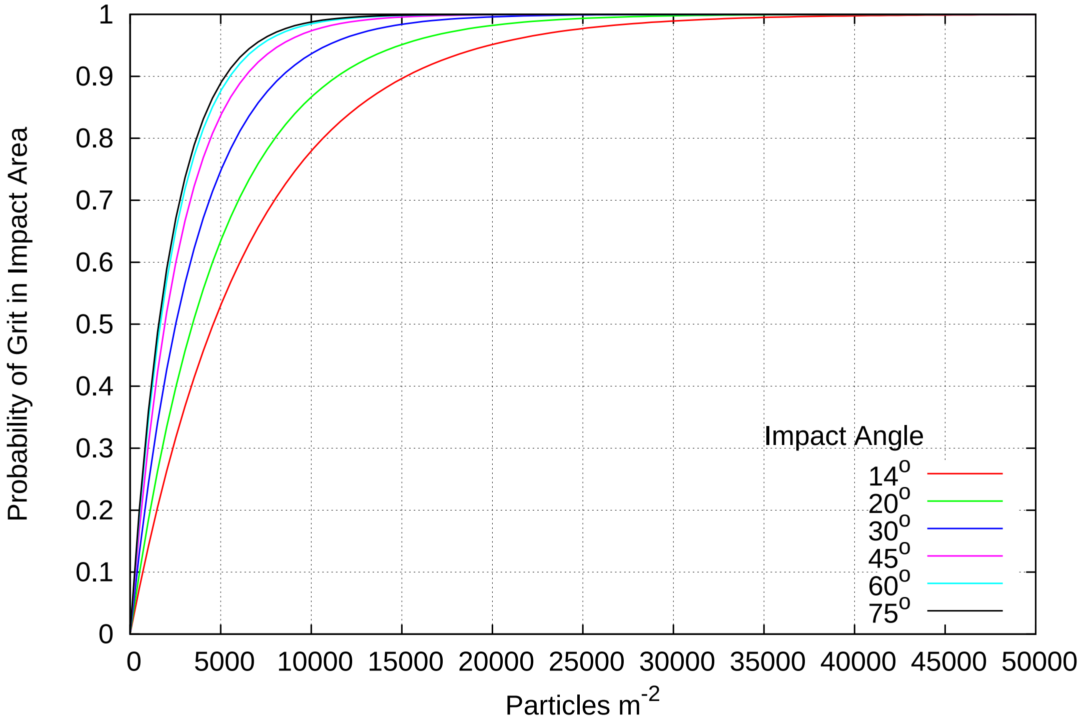

Figure 13: A plot showing the probability of impacting one or more particle, depending on impact angle, θ, and grit density, ρ. The fixed parameters were particle size, $r = 250\ \mu m$ and dropped from a height of 1.83 m (6 ft.)

ing was considered to be $p = 0.5$ for the grit-grit collision. Fig.(13) does not show much difference in the probability of impacting one or more particles with impact angle, this is expected since this probability depends on only the area and particle density. However, we expect the shallower impact angles to be more likely candidates for ignition since the drag length, L, is longer for these angles, providing more time for frictional heating to occur. Fig.(14) shows more complex behavior since the drag length strongly influences the probability of a grit-grit collision. It appears the worst case angle is $\sim 45^o$ with the probability of a collision falling off steeply with angles over 45^o.

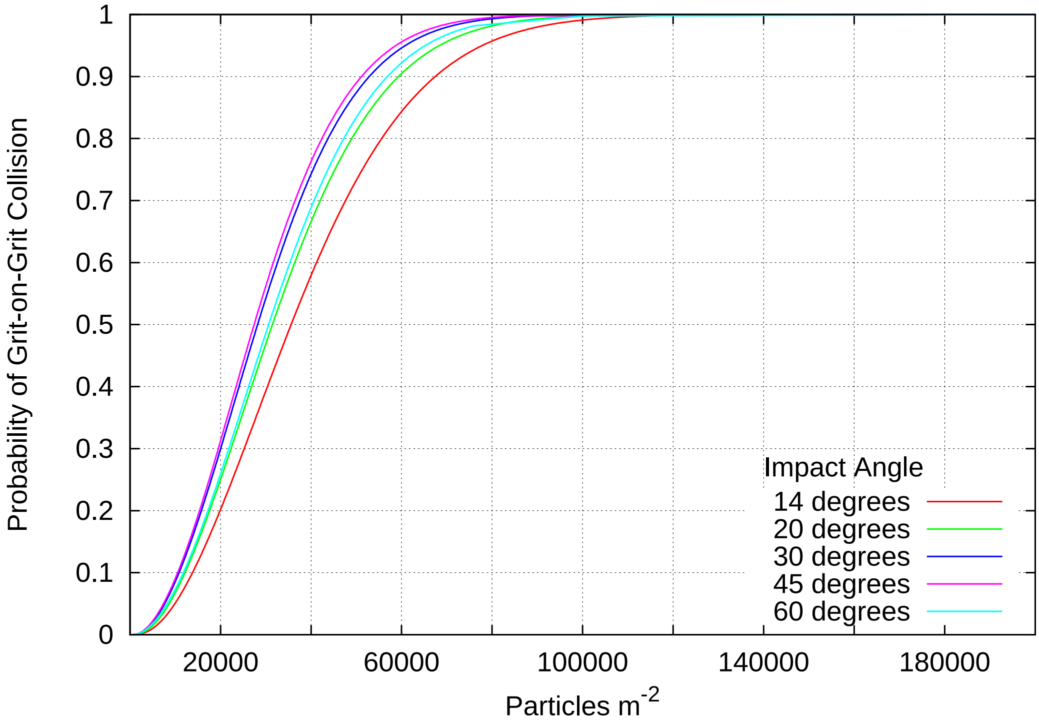

Figure 14: A plot showing the probability of grit-grit interaction depending on impact angle, θ, and grit density, ρ. The fixed parameters were particle size, $r = 250\ \mu m$, probability of moving, $p = 0.50$, and dropped from a height of 1.83 m (6 ft.)

6 Conclusion

In order to determine the cleanliness needed to mitigate the dangers of grit initiated ignition of explosives, two statistical quantities were derived. The first expression, Eq.(3), determines the probability that one or more particles are within the impact zone of the impacting explosive. This probability depends on the area, A, (which depends on the drop height and angle of impact) and the coverage density, ρ, of the grit particles on the target surface. The second quantity, Eq.(15), finds the probability that there will be a grit-grit collision, another mechanism which can lead to ignition. This expression depends on the area, A, swept out by the impacting explosive, the length of the skid, L, the density of grit particles, ρ, the radius of the grit particles, r, and the probability, p, that a particle will move with the impacting explosive or will remain stationary. Also, an expression for the maximum contact area is derived and fit to experimental data. This area depends on drop height, h, angle of impact, θ, the radius of curvature, R, and the experimentally fit ω. This allows one to easily calculate the above probabilities dependent on both drop height and coverage density, giving a measure needed for determining cleanliness standards and controls for drop height.

7 Notes

For a Python script to calculate the probabilities of interest please email Eric Heatwole at heatwole@lanl.gov or call (505)665-7897.

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